

A GAME THEORETIC ANALYSIS OF CONGRESSIONAL POWER DISTRIBUTIONS FOR A STABLE TWO-PARTY SYSTEM¹

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Attempts to make behavioral science a science often hinge on making vague terms precise or quantifiable, if possible. From game theory comes a definition of "power" (as applied to coalitions) in precise and quantifiable form. Once one has a quantitative definition of power, one wonders how this quantity will be distributed in a body of men with partially the same and partially opposite interests and under the influence of various other considerations. It is assumed here that the American legislative apparatus (involving both Congress and the President) is such a body. The picture is necessarily a drastically simplified one, but it has some essential and recognizable features of its referent: majority rule, the veto, the overriding of the veto, the two-party system, the defections of some members from one party to the other, the "die hards" who never defect. The results of rigorous analysis are in general in agreement with the "findings" of the political scientists about where the power resides under various conditions. But in addition this game-theoretical approach calls attention to other results, which are not often emphasized or realized. The method thus provides a framework for future enrichment and refinement of exact methods in political science.

THE theory of games (11) is a mathematical model for conflict of interest among intelligent and motivated agents; it is, therefore, not surprising that attempts are being made to apply it to some political science problems. So far, there are two such published efforts. The first consists of readings in game theory, assembled by Shubik (9), which serve to introduce some of the basic ideas and criticisms of the theory. The second, a paper by Shapley and Shubik (8),

is the first actual application of a portion of game theory to a political science problem. The present paper outlines another application, which is in a sense a natural successor to the Shapley-Shubik work.

We must emphasize that this is an outline, for we do not view the content as a serious attempt to study Congress as such; rather, it is a purposely oversimplified illustration of how a part of game theory may be applied to such studies.

To this end, we have taken pains to point out the nature of the assumptions made, the extent to which they are inherent in the model, and the extent to which they are simply matters of convenient exposition. We hold that a much more serious attempt must be made to abstract the central features of a legislature than we have undertaken before one can decide whether the particular model recommended is useful or not. In apparent

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contrast to our doubts that the formulation will sustain careful scrutiny by students of Congress, we have formulated a group of generalizations that summarize some of our detailed results and we have discussed these in the light of prevailing generalizations and some data. This should not be misinterpreted; it is only intended to demonstrate how it is possible to go from the mass of detail generated by the model to the types of generalization more familiar to a political scientist. We hope that our discussion will stimulate others to use these mathematical tools in deeper analyses of legislative bodies—studies in which data are related more closely to theory than we have done.

A legislature, as a voting body, can be viewed as having two inherently different aspects: a formal body of rules, called the legislative scheme, which establish the conditions under which a bill is passed, and the various peculiarities and limitations characteristic of a particular legislature working within the given scheme. Included in these limitations are such realities as the party structure, party discipline, the effects of pressure groups, etc. Following Shapley and Shubik, it appears that the scheme is usefully identified with one of the central notions in n -person game theory: the characteristic function. It is suggested that many of the "realities" of a legislature can be identified with another notion central to one of the several equilibrium theories. This theory is concerned with those couplings of a "power distribution" to a division of the participants into coalitions such that no changes occur. Since it is striking that the two-party system—a division of the participants into two disjoint coalitions—has remained stable for a long time, we shall assume that it is stable and inquire as to the theoretically necessary location of power in Congress and the presidency for such stability. We shall not attempt to discuss the much more profound question as to why the two-party system has evolved and why it is stable, but only what conditions are theoretically necessary on the power distributions in order that the system be stable. As will be seen, the theory is simply a formalization of the usual verbal discussions

about the location of power—a formalization that can be readily extended to more complicated models of Congress where many of us would find it difficult or impossible to extend a nonsymbolic analysis.

Our work relates to the Shapley-Shubik paper in the following way: They ignored any information one might have about specific legislatures and confined their attention to legislative schemes. Given such a scheme, they inquired into the possibility of a priori statements about the power distributions implied by the scheme. If one is willing to accept certain conditions as to the nature of such an a priori distribution, they establish that it is uniquely determined by the voting scheme. For the study of existing legislatures this is obviously insufficient and one must take into account some of the known realities and attempt to deduce from them other—known or unknown—assertions and to investigate the empirical truth of these consequences. Our purpose is to begin to deal with this problem.

Before turning to the details of the model, let us freely admit that to many—particularly to those with mathematical training—the following discussion will seem tedious and the content slight in relation to the length. It is true that the content could be contained in half a dozen or eight pages, but only at the expense of using more mathematics and thereby excluding as readers many of those we most want to address: political scientists concerned with the study of legislatures. One price of interdisciplinary communication seems to be length.

1. A mathematical representation of a legislative scheme

For the purposes of this paper, we shall suppose that the only function of a legislative body is to vote on bills which are presented to it; of course, this is not actually the case, but it may prove to be a suitable first approximation to a legislature and, at the least, it should be of interest to see what consequences can be drawn from it. For the moment, we shall not consider how a legislature may be divided into chambers nor how it may be partitioned into parties;

rather, we will simply think of it as a body of undifferentiated men who vote. Let us denote this set (the terms “class” or “collection” are also used) of men by the symbol L (which stands for “legislators”). Consider any subset S of the set L (for example, if L denotes the set of men in the United States Congress on January 1, 1955, then the Southern Democratic Senators form a well-defined subset of L). The rules of the legislative scheme under consideration must determine whether or not this set S is able as a voting coalition to pass a bill. Indeed, one way to prescribe the voting rules of the body is to list for each possible subset S whether or not it can pass a bill. Let us call those coalitions which can pass a bill *winning* and those which cannot *losing*.

The legislative scheme with which we shall be concerned is that of the United States. It involves two sets of men: Congress, which we shall denote by the symbol C , and the President, whom we shall denote by P . For our purpose, it will not prove necessary to take into account that Congress is divided into two houses, for we shall not be concerned here with the origination of bills, with committee activities, or with treaties, and we shall always assume similar majorities in both houses. A coalition in this scheme is winning if and only if either

1. it consists of a majority (in both houses) of Congress and the President, or
2. it consists of a two-thirds majority (in both houses) of Congress.

All other possible coalitions are losing. It should be noted that by ignoring the possibility of ties, we have made a minor idealization: this is not essential to the model and it can be eliminated at the cost of more *routine* labor later on.

Returning to the general case, we may assume that as a result of passing or defeating a bill, there are certain rewards accruing to those involved. These may range from outright money payments to individuals, through various forms of indirect compensation, to changes in relative prestige. Each of the individuals in the legislature is assumed to have a pattern of preferences among these outcomes. While it is very

difficult to ascertain these preferences in practice, they may still be postulated. If they are defined over all risky outcomes, i.e., outcomes consisting of chance distributions over the basic outcomes, and if they satisfy certain axioms, then the theory of utility establishes how the preferences may be represented by numbers (11, pp. 15–30). Since there is an extensive literature on this subject, brevity dictates that we cannot delve into it deeply. Nonetheless, it must be emphasized that the theory is controversial and that many authors do not feel that people can be expected to exhibit the consistency demand by the axioms. On the other hand, the axioms do have a certain compelling plausibility. One important difficulty in the theory, as it is now developed, is that the numbers representing preferences are not uniquely determined—the choice of both the zero and the scale unit is arbitrary. The important ambiguity is that of the unit, for it is impossible to say what changes in the underlying outcomes result in the same utility change for two different people. This is the famed problem of interpersonal comparisons of utility.

Assuming the existence of such utility functions and a solution to the problem of interpersonal comparisons, the theory of games shows how it is possible to derive a number for each logically possible coalition which represents, in utility units, the amount of the rewards that the coalition as a whole may insure for itself (11, pp. 238–243). The fact that interpersonal comparisons cannot now be made would seem to render this construction empty; however, it is not completely empty if one supposes there exists a solution to this problem and, in some contexts, it is possible to determine these summary numbers without actually obtaining the individual utilities (see below). In a sense, such a number associated with a coalition represents its power with respect to the single bill under consideration. When we use the word “power” in the rest of this paper, it shall mean only the numerical representation of rewards accruing to coalitions as evaluated by the members of these coalitions. It is important that no other

meanings of this word be read into our results.

The power, in the sense of the numbers just described, of a coalition depends not only on the ability of the coalition to pass a bill, but also on the individual evaluation of the outcomes; thus, we cannot anticipate in general that these numbers will meet very strong restrictions. In fact, in the theory of games they are shown to satisfy only two very reasonable requirements; these are discussed in the Appendix. Since our work is primarily illustrative of a mathematical method, we shall not attempt to deal with the completely general case, but rather we shall suppose that the individual evaluations of the rewards resulting from passing the bill under consideration are such that each of the winning coalitions has identical power and each of the losing coalitions has identical power. This, very clearly, is an idealization, but one which may be approximately true in some cases. It is not, however, essential to the methods we are illustrating, as is pointed out in the Appendix, so long as one can devise empirical methods for estimating the relative power of the various coalitions. As we shall see, it will not prove necessary to have these estimates for all possible coalitions, but only for a relatively few relevant ones.

With the assumption of the power equality of all winning coalitions and of all losing ones, there is no loss of generality in setting the power of a winning coalition equal to 1 and that of a losing one equal to 0; this we shall do. If S is any subset of L , we denote by $v(S)$ the power of S acting as a coalition, i.e., $v(S) = 1$ if S is winning and $=0$ if S is losing.

Of course, it is a theoretical fiction to speak of the power of a coalition. True, the power results from the collection of men acting as a coalition, and so in that sense it is associated with the coalition, but the rewards it represents must actually be rewards to individuals in the legislature. We cannot even say "rewards to just the members of the coalition which passes the bill," for the coalition may find it expedient to turn over some of the rewards to men not in the coalition. At least this is an a priori possi-

bility and though it will not actually occur under the assumption of equality of power for winning and for losing coalitions, with more general assumptions it can. We, therefore, suppose that the distribution of total power to the different legislators can be represented by a collection of n numbers x_i , where i is an index running over the n legislators. For example, if we number the legislators from 1 to n , then x_{10} denotes the power accruing to the legislator numbered 10. We shall stipulate that *all* the power is distributed to the legislators, i.e.,

$$x_1 + x_2 + \cdots + x_n = 1,$$

and that the smallest amount of power is 0, i.e.,

$$x_i \geq 0 \quad \text{for } i = 1, 2, \dots, n.$$

(In the vocabulary of the theory of games, such a distribution is called an imputation.)

We shall suppose, subject to some limitations to be given later, that during the pre-vote haggling and threatening each of the legislators is attempting to achieve as large a portion of this distribution of power as he can. The purpose of our analysis, among other things, will be to establish which, if any, of the distributions of power are in equilibrium in the sense that further haggling will not result in a modification of the distribution.

At this point, it is convenient to interrupt our pursuits to indicate what Shapley and Shubik have done in the paper mentioned earlier (8). Given a legislative *scheme*, as described by the winning and losing coalitions, they inquired what, if any, a priori statements could be made about power distributions. They quite consciously ignored all the specific information one might have or might obtain about a specific legislature, such as Congress at a certain date, and they concentrated entirely on the information given by the legislative scheme. We shall not attempt to reconstruct their argument, but we may mention the nature of the important and surprising theorem of Shapley's upon which their work rests. He has shown (7) that if an a priori distribution of power is required to meet some (apparently) weak and possibly acceptable conditions, then

there is a unique answer to the problem which can be expressed by a simple formula involving the numbers $v(S)$. The formula amounts to calculating for each legislator the chance that, if a coalition were built up successively by random selections from among the legislators, he would be the individual who converted it from a losing one to a winning one.

They presented several calculations of a priori power distributions for well-known voting schemes. For example, for the congressional-presidential scheme, the ratio of power of a single representative to the presidency is 2 to 350 and between a senator and the presidency the ratio is 9 to 350. If we take the House of Representatives and the Senate as single units, then they have equal a priori power and the presidency is two-fifths as powerful. If the congressional scheme were modified so that the presidential veto could not be overridden, then the House would have slightly less power than the Senate and the presidency would be about twice as powerful as either of them. As they point out, such a theory gives one a tool to examine the effect of revising legislative procedure, for "...it can easily happen that the mathematical structure of a voting system conceals a bias in power distribution unsuspected and unintended by the authors of the revision" (8).

Our purpose is to go beyond the Shapley-Shubik analysis of a legislative scheme to an analysis of a legislature. This will, of course, necessitate a model of what we mean by a legislature. In the next section, we shall present a model for Congress, which is illustrative of a class of models for legislatures. (The general class is discussed in the Appendix.) These models must attempt to capture some of the realities of specific legislatures—realities which are not part of the voting scheme. In the case of Congress, we mean by realities such facts as the party structure, the committee roles, the liberal-conservative dichotomy, the individual loyalties to party, personal animosities, etc. We shall not, by any means, attempt to deal with all of these in this illustrative example, but only with the party structure

and an approximation to party loyalty. This will permit a plausible first approximation which is sufficiently simple to render the analysis fairly transparent. The effect of introducing other factors is only to increase the details of analysis without modifying the basic procedure.

2. A model of Congress

We shall start with the fact that every legislator is identified with one of two non-overlapping political parties. We shall assume for simplicity, and with little practical loss of generality, that whichever party has a majority in one chamber of Congress has a majority in the other. Let us label the majority party as number 1 and the minority party as number 2, and let us denote by C_1 the set of congressmen in the majority party and by C_2 the set in the minority party.

An arresting fact about Congress is the stability of the two-party system, i.e., the simple fact that it has not split into more than two parties or reduced to one. One of the major questions to which we shall address ourselves is whether there exist distributions of power for our model of Congress which permit a stable two-party system. Thus, if the President is a member of the majority party, we shall be interested in the stability (in a sense yet to be defined) of the partition of the voting body (which includes the President) into C_2 on the one hand and C_1 plus P on the other. Let the set consisting of C_1 and P be denoted by $C_1 \cup P$. Let τ_1 denote this partition, $(C_1 \cup P, C_2)$. Similarly, if the President is in the minority party, we shall be interested in the partition

$$\tau_2 = (C_1, C_2 \cup P).$$

While the two-party structure is known to be stable, it is equally clear that some, if not most, bills are passed by coalitions different from the party coalition. The conservatives of the two parties may join as a temporary coalition to pass a single bill without causing the disruption of the party structure. However, given a particular bill and a particular Congress, there are certain coalitions which could not conceivably form.

If we restrict our attention to the partitioning of Congress by parties, these limitations on the formation of coalitions (to pass a bill) are, therefore, limitations on defections from the parties. Such limitations are produced by a number of factors such as pressure from constituents, party discipline, pressure from lobbies, the particular issue at stake, etc. However they may be generated, they can be described in terms of the set of Congressmen who can be induced to defect to the other party. Here we make two simplifying assumptions: first, that there are no abstentions from voting and, second, that the only defections are from one party to the other. Actually, the second of these omits the important possibility of defectors from both parties joining to form a winning coalition. Again, as will be seen, there is no inherent reason for making this assumption: it only reduces the amount of routine calculation and the amount of space needed to present the results.

Because of the nature of the legislative scheme, there are only two groups of defections which are of interest: a defection which swells the ranks of the other party to a two-thirds majority in each house, in which case a presidential veto can be overridden, and a defection which fails to achieve a two-thirds majority in at least one house but does result in a simple majority in each, in which case a coalition of the majority and the President can pass the bill. In either case, the party in question theoretically will only be interested in defections which just produce the desired majority—all other votes are technically superfluous. Now, at the time of any given vote, it does not seem too implausible to suppose that in principle the potential defectors in each party can be graded from the most to least willing to defect from their party. If so, and if the defectors are added to a coalition in order of decreasing willingness to defect, then there is a unique set of defectors which will just create a simple majority, if that is possible, and a unique set which will just create a two-thirds majority, if that is possible. So, when we speak of a set of defectors which create a certain majority, we shall mean that mini-

mal set of congressmen, drawn from among those most willing to defect, which is just necessary to form the majority.

With this assumption, then, we may divide the congressmen of parties 1 and 2 into two non-overlapping sets: C'_1 and C'_2 will denote the sets of *potential defectors* from parties 1 and 2 respectively and C''_1 and C''_2 the remaining members of each party, who will be called the *diehard*s.

There are a number of different cases which can occur in this idealized Congress. Since there is no a priori reason to exclude any of these we shall examine each of them separately, and on the basis of this exhaustive examination, we shall draw some general qualitative conclusions from the model (section 5).

First, there are four basically different partitions of Congress into the two-party system: either party 1 (which by assumption has a majority) fails to have a two-thirds majority in at least one chamber, or it has a two-thirds majority in both, and either the President is a member of party 1 or of party 2.

Second, there are twelve different possible limitations on coalition changes from the two-party partition. These twelve arise from a selection of one of the alternatives in each of the following three classes of alternatives:

1. either the President is (or feels) free to defect from his party, or he is not;
2. party 1 plus the defectors from party 2 either form only a simple majority, or they form a two-thirds majority in both houses; and
3. party 2 plus the defectors from party 1 either fail to form a simple majority in at least one house, or they form a simple majority in both houses but not a two-thirds majority in at least one, or they form a two-thirds majority in both houses.

We observe that not all these limitations are compatible with all the coalition partitions, e.g., it is not possible for the majority party to have a two-thirds majority in both houses and for the addition of defectors from the other party to reduce this to a

simple majority. If such cases are excluded, then there are a total of 36 cases to be examined.

3. Equilibrium power distributions in a special case

To introduce the equilibrium notion we shall use and to illustrate the typical calculations involved, we shall choose one of the 36 cases; it matters little which one we take. Let us suppose the President is in party 1 and that party 1 has only a simple majority in Congress. Thus, we shall be concerned with the partition $\tau_1 = (C_1 \cup P, C_2)$, where C_1 does not form a two-thirds majority. As the system of limitations on changes from τ_1 , let us choose the case where

1. The President is free to defect;
2. party 1 plus the defectors from party 2 form only a simple majority; and
3. party 2 plus the defectors from party 1 form only a simple majority.

Now, from these assumptions, we see that a two-thirds majority does not exist among the possible changes; thus any winning coalition that can form must include the President. It is easy to see that there are only three such coalitions:

$$C_1 \cup P, C_1 \cup C'_2 \cup P, \text{ and } C_2 \cup C'_1 \cup P,$$

where, as before, if A and B are sets, $A \cup B$ is the set whose elements consist of those in A and those in B . One of these three will have to form to pass the bill; we shall not concern ourselves with which (there are certainly not enough assumptions even to begin to answer that question), but rather with the location of power in such a Congress under the assumption that the two-party system is stable.

Consider a power distribution $x_i, i = 1, 2 \dots n$, which is offered as being compatible in this Congress with the two-party partitioning of Congress. If any legislator i not in $C_1 \cup P$ has $x_i > 0$, then we know that the sum of the x_j for j in $C_1 \cup P$, must be less than 1. But since $C_1 \cup P$ is winning it can command power of 1, and so it can form and each of its members can receive more than they did in the arrangement $x_i, i = 1, 2, \dots n$. Since we are assuming

each legislator is out to better his gains of power, we must conclude that if anyone outside $C_1 \cup P$ has power, then the combination of the two-party partition and the given power distribution cannot be in equilibrium. A similar argument applies to $C_1 \cup C'_2 \cup P$ and to $C_2 \cup C'_1 \cup P$, but not to any other set of legislators for either they are losing, and so can offer no power gains, or they are not among the admissible changes. Thus a power distribution $x_i, i = 1, 2 \dots n$, is in equilibrium with τ_1 only if $x_i = 0$ for any legislator i not in *each* of the three coalitions. Thus, the power must be distributed over those who are in *all* three of the winning coalitions, i.e., over $C'_1 \cup P$. In other words, in this situation the power is distributed over the defectors from party 1 and the President. Exactly how it is distributed over these men is not determined in this simple model: it presumably depends upon factors which we have not taken into account. If we were to extend the model to include such things as the actions of committees, the origins of the bill, and so on, we could then expect a much more detailed determination of the power distributions.

4. The equilibrium power distributions for the two-party system

In order that there shall be no ambiguity in our definition of equilibrium, we shall be somewhat more formal than we have been up to now. Let $v(S)$ denote the number representing the power accruing to the set S if it forms a voting coalition. Let X stand for the distribution of power $x_i, i = 1, 2, \dots n$. Let τ denote a partitioning of the legislature into coalitions (while we have assumed partitions into only two coalitions, this assumption is by no means necessary). Let the symbol $\psi(\tau)$ stand for the class of coalitions which can form, if there is any reason to do so, when the legislature is partitioned according to τ . Then, we shall say that the pair (X, τ) is ψ -stable (which is simply the name assigned to this particular definition of equilibrium) if for each set S in $\psi(\tau)$.

$$v(S) \leq \sum_{i \text{ in } S} x_i.$$

(The last symbol, $\sum_{i \in S} x_i$, simply means the sum of the values x_i for each legislator i in the coalition S .)

A re-examination of the analysis of the special case discussed above shows that it is just a special case of this definition.

Actually, the definition we have given here is slightly different from that presented in the mathematical literature (3, 4). There it was stipulated that if a legislator receives no power at all, i.e., if $x_i = 0$, then he shall

not be involved in a coalition with any other legislators. The argument for imposing this condition arises simply from asking why he should cooperate with others if this cooperation does not result in any gain for him. This is a powerful argument in situations which occur only once, but it seems less convincing in a legislature where one bill is but part of an on-going process.

Now, while we have in fact been ignoring the on-going process (see the Appendix),

TABLE 1
THE POWER DISTRIBUTIONS FOR A STABLE TWO-PARTY SYSTEM UNDER THE GIVEN CONDITIONS

No.	Presidential Defection	Party of President	Size of Party 1 Majority	Size of Party 1 plus Party 2 defectors	Size of Party 2 plus Party 1 defectors	Locations of Power		
1.a.i ii iii	Possible	Either	simple	simple	less than majority simple two-thirds	Party 1, President Party 1 defectors, President Party 1 defectors		
1.b.i ii iii				two-thirds	less than majority simple two-thirds	Party 1 Party 1 defectors Party 1 defectors		
2.a b c				two-thirds	less than majority simple two-thirds	Party 1 Party 1 defectors Party 1 defectors		
3.a.i ii iii			Not Possible	1	simple	simple	less than majority simple two-thirds	Party 1, President Party 1, President Party 1 defectors
3.b.i ii iii						two-thirds	less than majority simple two-thirds	Party 1 Party 1 Party 1 defectors
4.a.i ii iii					2	simple	simple	less than majority simple two-thirds
4.b.i ii iii	two-thirds	less than majority simple two-thirds					Party 1, Party 2 defectors Party 1 defectors, Party 2 defectors Party 1 defectors, Party 2 defectors	
5.a b c	Not Possible	1	two-thirds	two-thirds	less than majority simple two-thirds	Party 1 Party 1 Party 1 defectors		
6.a b c		2	two-thirds	two-thirds	less than majority simple two-thirds	Party 1 Party 1 defectors Party 1 defectors		

we shall at least take it into account to the extent of waiving the second condition. It may also be worth noting that from the mathematical point of view this does not constitute a very serious change in the definition for the second condition has not played an important role in most of the theorems proved.

We may now give (Table 1) the power distributions which coupled with the party partitioning of Congress are stable in each of the 36 cases described in Section 2. The conclusion in each case arises by an argument like that given in Section 3 (that case is number 1.a.ii in Table 1).

5. Conclusions

By examining the summary of results in Table 1, certain generalizations implied by the model are clear. We may list six which seem interesting:

1. In all cases the arrangement of Congress into two opposed party coalitions is stable provided the power is distributed as indicated. In very many cases, however, it is necessary to form coalitions other than along party lines in order to produce a winning coalition, i.e., to pass a bill. In only one case (4.a.i) are the limitations so stringent that no working majority can form: this is when the President is of the minority party and will not defect to the majority, the majority party has only a simple majority even with the defectors from the minority, and the minority does not have a simple majority even with the defectors from the majority. What is interesting is that in only one case of the 36 can such an impasse result.

2. In all circumstances, the President is weak when the majority party—whether he is a member of it or not—has a two-thirds majority. If this model has any relation to reality, we must conclude that a President should fear a real Congressional landslide for *either* party.

3. The President possesses power (from voting considerations) only when neither party can muster more than a simple majority even with the help of the defectors from the other party.

4. The only circumstances when the

minority party is the holder of any power is when the President is in the minority party and he is unwilling to defect to the majority.

5. Under all conditions, if the defectors from party 1 added to party 2 fail to form a majority, then the diehards of party 1 possess power. The only other case in which they possess power is when the President is a member of party 1, he is unwilling to defect, and party 2 plus the defectors from party 1 form only a simple majority (cases 3.a.ii, 3.b.ii, and 5.b).

6. The only case when the party 2 diehards possess any power is when the President is a member of their party, he is unwilling to defect, party 1 has only a simple majority, and party 2 plus the defectors from party 1 form either a majority or a two-thirds majority (cases 4.a.ii and iii).

In connection with these last two statements, we note that there are a large number of situations where the diehards are not in a position to command power in a stable two-party system, and those situations in which they do have power appear to be ones not likely to occur often in practice. Recall that when we introduced the concept of a ψ -stable pair (section 4) we noted that in the original mathematical definition it was stipulated that a nontrivial coalition would not exist if its members did not benefit from their participation in the coalition. While we waived this condition for our work in this paper, there is still some force to the argument if we think of the long-term existence of a legislature. Thus, if voting coalitions tend to stabilize in time and if consideration is not given to the diehards by the other party members, it should not be too surprising to find a sudden change in their behavior. Of course, it must be understood that this is not a conclusion from the model but an extrapolation, for the model (as a special case of game theory) is inherently static and does not attempt to deal with such changes in time.

Despite the various limitations of the model, which we have already noted, the generalizations we have drawn from it do not appear to be seriously inconsistent with a number of political science “findings”

concerning Congress, and in several cases they appear to emphasize aspects of congressional power which may not have been given adequate attention. The sorts of statements with which the model is not in serious contradiction are:

The President, as a legislator, is weak when either

1. his party is in the minority;
 2. his party is in the majority but is not committed to his program; or
 3. either party can muster sufficient strength to overturn a veto:
- and the President, as a legislator, is strong when either
4. his party is in the majority and is committed to his program;
 5. his party is in the majority, whether or not it is committed to his program, provided that it cannot muster sufficient strength to overturn a veto;
 6. supporters from his own party and defectors from the opposition party constitute a pro-administration majority coalition.

Such statements as these are not in all cases fully acceptable generalizations from the model, e.g., the first one must be qualified by the condition that one or another congressional coalition has a two-thirds majority, for if not the President possibly does possess power (see cases 1.a.i, 1.a.ii, 3.a.i, 3.a.ii, and 4.a.ii). A careful examination of our results will show that several others of these statements need some qualification before one can say that they coincide with the results of the model, but the spirit of them is much the same. It would be of some interest to know how stable these generalizations are under slight changes in the assumed limitations on coalition changes (as given by $\psi(\tau)$): for example, one might examine the effect of including as possible coalitions those made up from the defectors of the two parties. This would result in many more cases to examine, but none would be any more difficult to deal with than those above.

Let us emphasize once again that both these statements and the generalizations arrived at using game theory refer only to one aspect of Executive-Legislative rela-

tions. They do not deal with the crucial power position of committees and party leaders, the special position of the Rules Committee in the House, the filibuster power in the Senate, or a number of other important factors. Consequently, no claim can be made that our generalizations would not be substantially modified were the model refined.

On the other hand, they do emphasize several factors which are sometimes neglected or which are occasionally minimized in political science literature. One of these is concerned with the power position of diehards vis-à-vis defectors. As V. O. Key, Jr.² and James M. Burns, (1, see also 2) among others, have noted, party defectors often constitute either the effective working majority or the effective opposition, irrespective of whether or not they are members of the nominal majority party. The model suggests that in only a relatively few (and relatively unlikely) cases do the party diehards possess effective power. In the majority of the 36 cases, party defectors hold the balance of power.

This observation suggests that more attention needs to be paid to the analysis of Congress not in terms of nominal party majorities or minorities but in terms of cross-party groupings which might be tentatively classified as we have done. Thus far, empirical research does not provide a clear answer to the question of the extent of party cohesion,³ let alone answers to

² "In the American Congress the weak ties of the majority encourage the minority to wean away followers from the majority party and to determine the outcome of at least some and at times many legislative issues. It is not uncommon for the working 'majority' to be composed of a substantial part of the minority plus a sector of the nominal majority . . . the genuinely effective 'opposition' often consists, not of the minority, but of recalcitrant members of the majority who hold a balance of power within the House or Senate" (6, p. 706).

³ The failure is reflected in the differing views held of the matter. Thus, Burns' view that "Party cohesion is still slight today," (1, 40) confirming A. Lawrence Lowell's analysis of fifty-four years ago, is similar to that advanced in the American Political Science Association's report "Toward a More Responsible Two-Party System." But a diametrically opposite position has been taken by a number of other political scientists. See for example, (10).

such questions as: who are the diehards and defectors, from what areas of the country do they come, why do they operate as such, and is the theoretically greater power of the defectors reflected in the legislation which finally gets to a vote, in committee assignments, or in other kinds of prestige and influence?

Our game theory generalizations also point up a number of inherent features of Executive-Legislative relations. It is clear from the model that the President, under the prevailing system of loose party discipline, does not necessarily gain when his party is returned to Congress with an overwhelming majority. Although most students of legislative behavior are of the view that the party of the President should have the main responsibility for the organization of Congress, our analysis suggests that the President is not much advantaged when his party elects two-thirds or more of the Senators and Representatives. Other things being equal, the President's power as a legislator appears to be great when his own party controls between 51% and 66% of Congressional seats, and when no cross-party combination of diehards and defectors can total more than two-thirds of the membership. Tactically, this means that his party should have only about 55% of the seats.⁴

Rather paradoxically, the President's power can also be great even when his own party is in the minority, provided that he is still in control of a bloc of his party possessing at least 35% of the Congressional votes. For in such cases, and so long as the 35% is loyal to the party chief, the majority

⁴ These generalizations are not contradicted by the record of Presidential vetoes cast during the Roosevelt Administration. As the following table shows, 69% of the vetoes cast by Roosevelt were cast during the three Congresses in which one or both houses were controlled by a two-thirds or better Democratic majority.

President	Year	Congress	House		Senate		Vetoes
			D	R	D	R	
Hoover	1931-33	72nd	220	214	47	48	18
Roosevelt	33-34	73rd	310	117	60	35	73
Roosevelt	35-36	74th	319	103	69	25	148
Roosevelt	37-38	75th	331	89	76	16	117
Roosevelt	39-40	76th	261	164	69	23	167
Roosevelt	41-42	77th	268	162	66	28	79
Roosevelt	43-44	78th	218	208	58	37	45
Roosevelt	45-46	79th	242	190	56	38	76
Truman							

(and opposition) party can initiate legislation, but it cannot subdue the veto power. The President, therefore, holds effective power as either party diehard or defector; in the former role, he can bring about a legislative stalemate, and in the latter role, he can transform an opposition which is impotent, so far as positive power is concerned, into a governmental party.

In closing, we must once again return to the limitations of the present version of the model. Whether or not this game theoretic analysis can provide us with any illumination of the Congressional power structure depends very largely on its potentialities of refinement to include factors which are known to be of importance. Thus, the question reduces to our ability to determine the power of coalitions, $v(S)$, and the limitations on coalition change, $\psi(\tau)$, for more complex real situations. More sophisticated models may very well necessitate the collection of empirical data in an attempt to determine these quantities. Presumably, the limitations on coalition defections, etc., can be assessed from obtainable data, in which case there seems to be no reason to assume, a priori, that the model must remain in its present elementary state. Equally, there seems no reason to suppose, if the generalizations deduced from the present model are at all interesting, that future refinements will not produce results of similar but more subtle interest.

Appendix

The purpose of this appendix is to indicate a little more explicitly the general form of the game model which we have illustrated by a special case in the main body of the paper. As we have indicated in several places, we believe that the methods discussed here are applicable to more refined models of legislatures; however, such applications surely will be more tedious in labor and will, in all probability, require the collection of empirical data.

As we said, game theory begins with a situation consisting of possible choices by individuals and individual outcomes associated to the choices. The individuals are assumed to have patterns of preferences among the outcomes which meet the axioms of utility theory, and so their preferences can be represented numerically. From this structure, the theory of games establishes that to each subset S of the individuals

there is assigned a number $v(S)$, which is calculated from the individual preferences. While in the model we examined $v(S)$ was either 0 or 1, in general this is not the case. If, for example, different rewards or different evaluations of the rewards occur for passing a bill, then the power of different coalitions which could pass the bill will be different. Or if we do not focus our attention on one bill, but on a series of bills, then since it is unreasonable to suppose that they will all result in the same rewards, we must suppose that the values $v(S)$ will depend upon more than the ability of the coalition S to pass a bill. The task of estimating $v(S)$ is clearly an empirical problem of some magnitude; there can be no doubt that it is a major stumbling block to the successful application of these methods. In particular, the theoretical construction of passing from individual preferences to $v(S)$ cannot be used, except in very special circumstances, to calculate $v(S)$; the practical difficulties are too great. More direct procedures are needed. One which may be useful are educated guesses made by well-informed observers. Another is a proposed, but untried, technique closely related to the methods of utility theory but applied directly to the coalitions and not to the individual outcome; this is discussed by Luce and Adams (5).

The collection of numbers $v(S)$, for all subsets S of the given set L (of legislators), form what in mathematics is called a real-valued set function v . This construct of game theory is called the *characteristic function* of the game. It might appear that since there is wide latitude in the individual evaluations of outcomes which lead to the characteristic function v , there can be no constraints on v in general. While this is almost true, it can be shown that it must meet two conditions, but in general no others can be established. Let us suppose that R and S are two subsets of L which have no legislators in common, and let $R \cup S$ be the subset consisting of the legislators in R plus those in S . Then it can be shown that

$$v(R \cup S) \geq v(R) + v(S).$$

In words, this simply means that the coalition $R \cup S$ can do everything R and S can do as separate coalitions, and possibly more. The whole is at least as great as the sum of its parts. The second condition, while mathematically significant, certainly appears trivial when one thinks of a legislature. It says the set ϕ which has no members—the empty set—has no power:

$$v(\phi) = 0$$

The interesting one is the former. In the case of the model we were discussing, it is particularly easy to see that it holds. If R and S are both losing $v(R) = 0$ and $v(S) = 0$. But $R \cup S$ may be either winning or losing, so the inequality holds. If R is winning and S is losing (or equally, R losing and S winning), then since $R \cup S$ is a set which includes the winning set R , it too must be winning. Thus, $v(R) = 1$, $v(S) = 0$ and $v(R \cup S) = 1$, and so equality exists. Finally, consider the case where both R and S are winning. Recall that we said R and S have no legislators in common, so we have the situation that if R passes a bill A which the coalition S does not like, then S can bring the negative of A before the legislature. Since S is winning it can pass not- A . Thus, the legislature would reach the deadlock where it could pass both A and not- A ; clearly, no acceptable legislative scheme would have this property, so the assumption that both R and S are winning is not meaningful. Hence, we conclude that the inequality must be met.

As in the special model, the equilibrium states of the legislature, no matter what characteristic function is involved, are given in terms of a pair (X, τ) , where X is a distribution of power to the individuals and τ is a partition of the legislature into coalitions. The distribution of power, or imputation as it is called, X must satisfy two conditions. It is a partitioning of the total power available:

$$x_1 + x_2 + \cdots + x_n = v(L).$$

and no individual receives less power than he can be certain of getting:

$$x_i \geq v(i) \quad \text{for } i = 1, 2, \dots, n.$$

Finally, the model supposes that the realities of the situation, insofar as they produce limitations on shifts from one partitioning of the legislature to another, can be given in the following way. For each partition τ one is interested in, there is given a list of admissible coalition changes, which we denote by $\psi(\tau)$. It will be recalled that where τ was taken to be the partition into parties, $\psi(\tau)$ involved coalitions formed through defections from one party to the other. Again, it is an empirical problem to estimate these limitations. This problem will become particularly grave when an attempt is made to refine the model to take into account committees and other relevant features of Congress. Yet students of Congress have discussed such limitations and it may not be impossible to employ careful statistical studies to gain the necessary detailed data.

The application of the model is straightforward if the two empirical problems can be overcome. First we choose a partitioning τ of the legislature which we wish to investigate (possibly along party lines, but not necessarily). Second we determine $\psi(\tau)$. Third for each subset S in $\psi(\tau)$, we determine $v(S)$. Observe that so long as we deal with only one τ , it is not necessary to determine the whole of the characteristic function, but only the values for the admissible coalitions as given by $\psi(\tau)$. Fourth, we determine those distributions X which with τ are ψ -stable using the definition given in section 4. The fewer coalitions that are admissible, the less precisely defined is X , the more that are admissible, the more precisely defined is X . In many games, if too many coalitions are admissible, there is no X which meets all of the conditions of the definition of ψ -stability, in which case one is forced to conclude that the partition τ is not stable. The effect of the limitations given by ψ is to increase the stability of a legislature, or in general, of a game.

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Laws and institutions must go hand in hand with the progress of the human mind. As that becomes more developed, more enlightened, as new discoveries are made, new truths disclosed, and manners and opinions change with the change of circumstances, institutions must advance also, and keep pace with the times. We might as well require a man to wear still the coat which fitted him when a boy, as civilized society to remain ever under the regimen of their ancestors.

—Thomas Jefferson